Go4Hybrid

Grey Area Mitigation for Hybrid RANS-LES Methods

Status at NLR
Johan Kok

Non-zonal Grey Area Mitigation approaches

- Reduction of SGS stresses:
  - baseline: High-Pass Filter (HPF) SGS model
  - alternatives:
    - recursive high-pass filter (Butterworth)
    - Vreman and Nicoud algebraic eddy-viscosity models
- Triggering of instabilities:
  - baseline: stochastic eddy-viscosity SGS model
  - alternative:
    - stochastic model of energy backscatter
    - temporal and spatial correlation of stochastic terms
- All tested for test case F2 Spatial Shear Layer
  - X-LES (k-ω based DES)
  - fixed RANS-LES interface at trailing edge for testing only
  - coarse grid: 1.3 M cells

High Pass Filter

- Reduction subgrid stresses: $\tau \sim \nu_t S$
- Existing HPF SGS model:
  - compute SGS stresses from velocity fluctuations u'

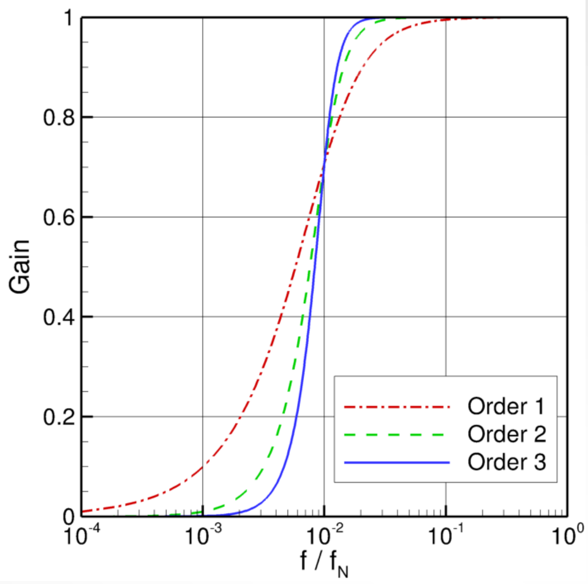
$$\tau_{ij} = 2\nu_t\left(S'_{ij} - \tfrac{1}{3}\partial_k u'_k \delta_{ij}\right) - \tfrac{2}{3}k\delta_{ij} \quad , \quad \text{if } l > C_1\Delta$$

$$u'(x,t) = u(x,t) - \tfrac{1}{t}\int_0^t u(x,s)\mathrm{d}s$$

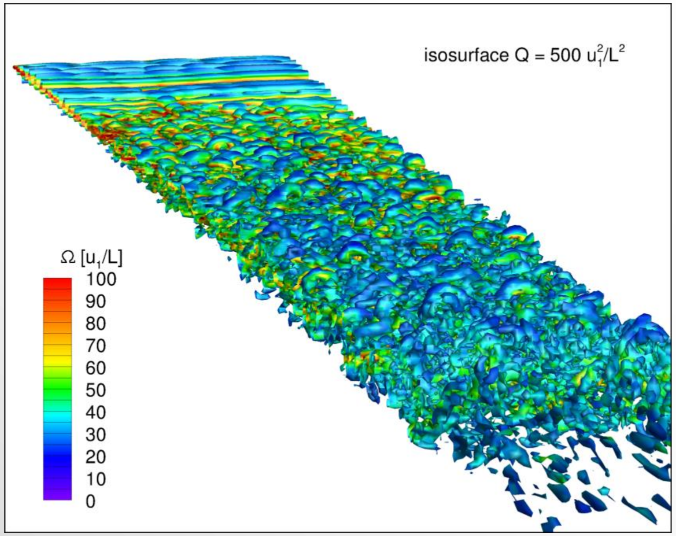
  - this high-pass filter has some limitations in applicability

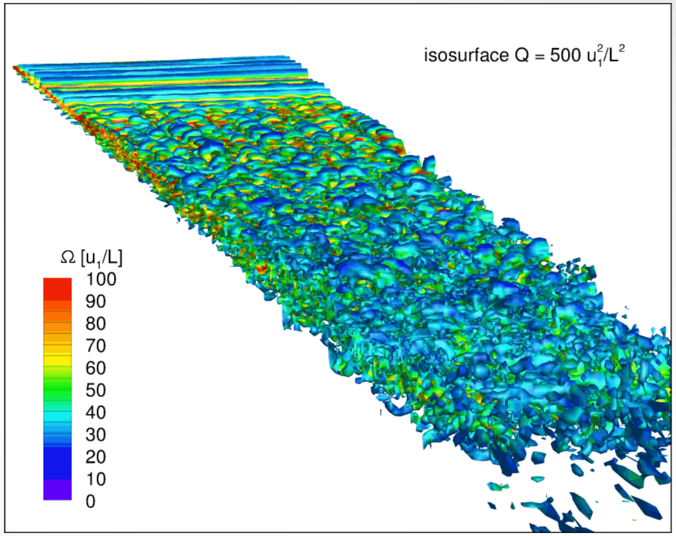
- Alternative 1: Butterworth-type filter
  - filtering of frequencies below certain cut-off frequency
  - recursive definition

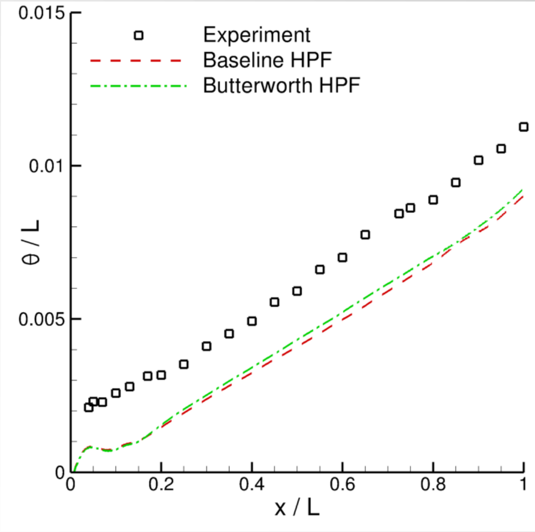
$$\Sigma_{k=0}^{P} a_k u'(t_{n-k}) = \Sigma_{k=0}^{Q} b_k u(t_{n-k})$$



Spatial shear layer: Alternative HPF SGS model







Baseline HPF
(+ stochastic)

Butterworth HPF
(+ stochastic)
1st order
$f_c = 10\, u_1/L = 415.4\ \text{Hz}$
$(T_c = 0.1\ \text{CTU})$

Momentum
thickness

Reduction of eddy viscosity

- Reduction subgrid stresses: $\tau \sim \nu_t S$
- Alternative 2:
  - reduce eddy viscosity using algebraic SGS models
  - Vreman model (2004) and Nicoud σ model (2011)

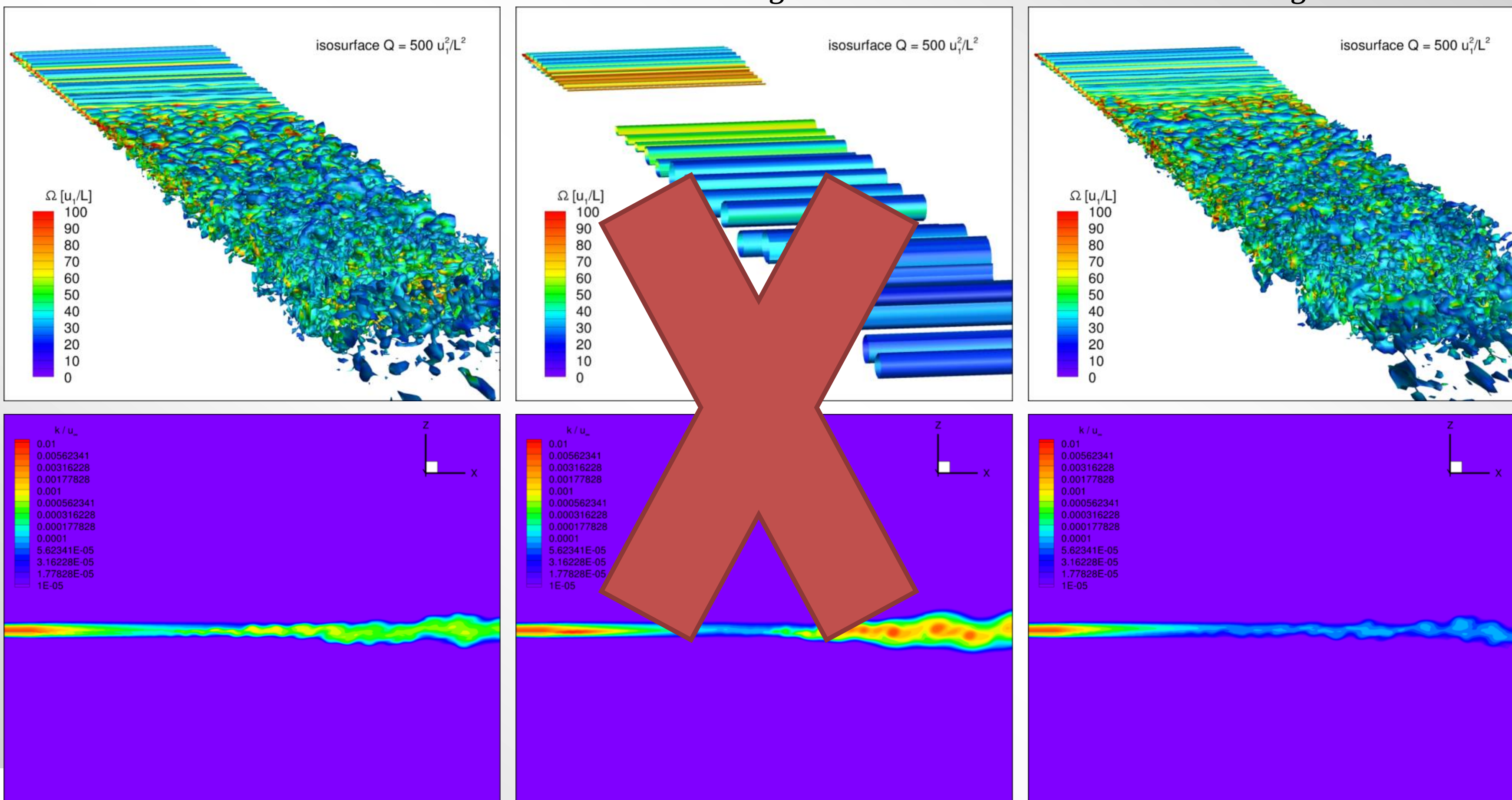
$$\nu_{sgs} = \left(C_{sgs}\Delta\right)^2 D_{sgs}(u)$$

  - $D_{sgs}(u)$ determined by invariants of $G = (\nabla u)^T(\nabla u)$
  - zero eddy viscosity for pure shear (Vreman) or for nominally 2D flow (Nicoud σ)
  - same approach as CFDB: obtain Vreman or Nicoud model when balance between production and dissipation in k-equation

$$P_k = \nu_t D^2 \quad , \quad D = \begin{cases} S, & \text{if } l \leq C_1\Delta \\ \sqrt{\beta_k}\left(\frac{C_{sgs}}{C_1}\right)^2 D_{sgs}(u) , & \text{if } l > C_1\Delta \end{cases}$$

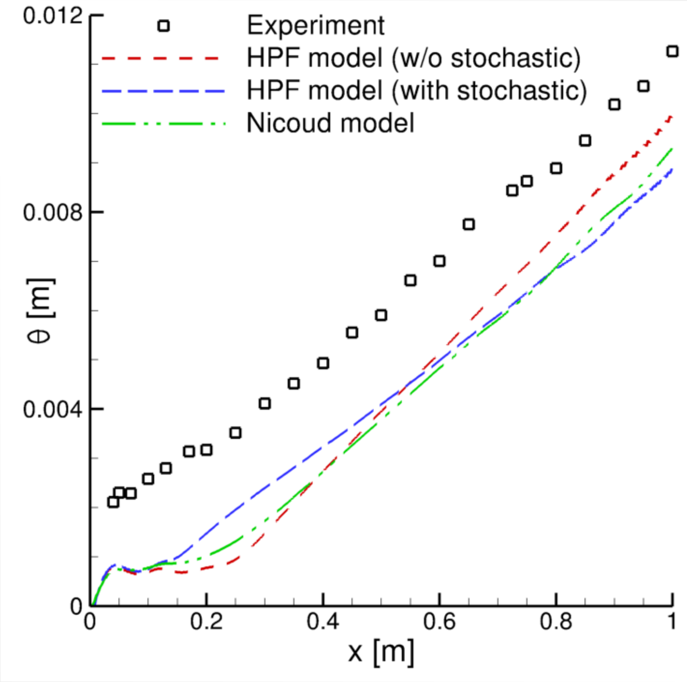
Spatial shear layer: Vreman and Nicoud models

Baseline HPF | Vreman ($C_{sgs} = 0.28$) | Nicoud σ ($C_{sgs} = 0.45$)



(All without stochastic model)

Spatial shear layer: Nicoud σ model



Triggering of instabilities

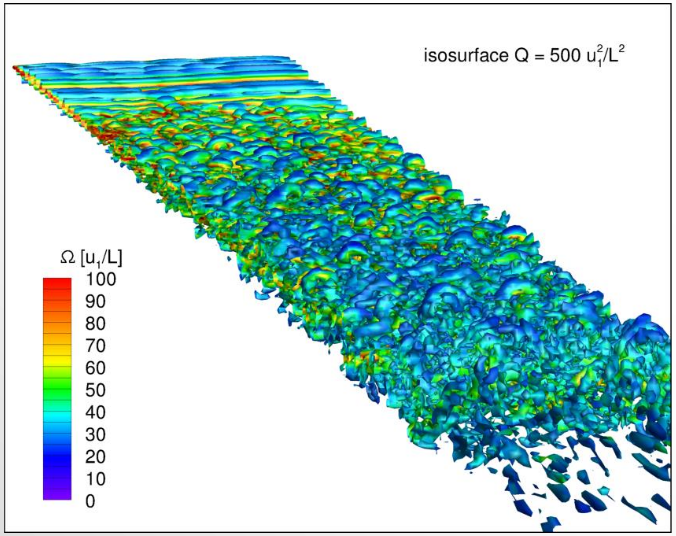
- Existing stochastic model:
  - modification of eddy viscosity with random variable $\xi = \mathrm{N}(0{,}1)$
    $$\nu_t = \xi^2 C_1 \Delta \sqrt{k} \quad , \quad \text{if } l > C_1 \Delta$$
  - crude approach
  - less effective when combined with HPF SGS model
- Alternative stochastic model: modelling of backscatter
  - based on models of Leith (1990) and Schumann (1995)
  - independent of HPF SGS model

$$\tau_{ij} = 2\nu_t\left(S_{ij} - \tfrac{1}{3}\partial_k u_k \delta_{ij}\right) - \tfrac{2}{3} k \delta_{ij} - R_{ij} \quad , \quad \text{if } l > C_1 \Delta$$

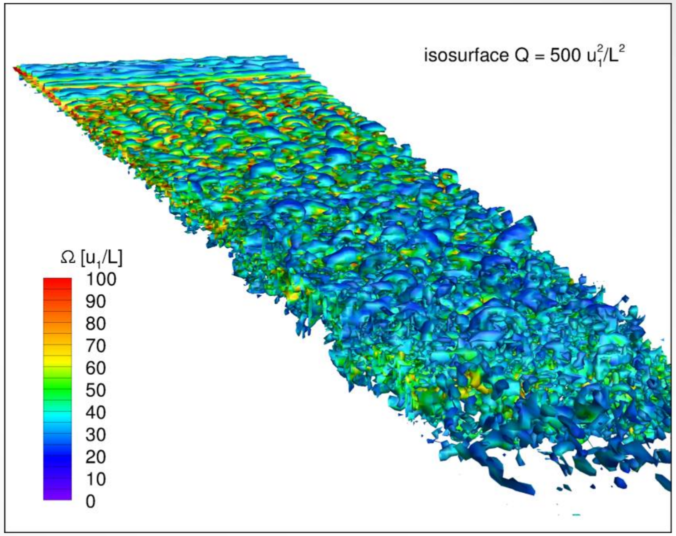
$$\nabla \cdot \boldsymbol{R} = \nabla \times (C_L k \boldsymbol{\xi}) \quad , \quad \xi_k = \mathrm{N}(0{,}1)$$

  - $\nabla \cdot \boldsymbol{R}$ is solenoidal: does not function as noise source

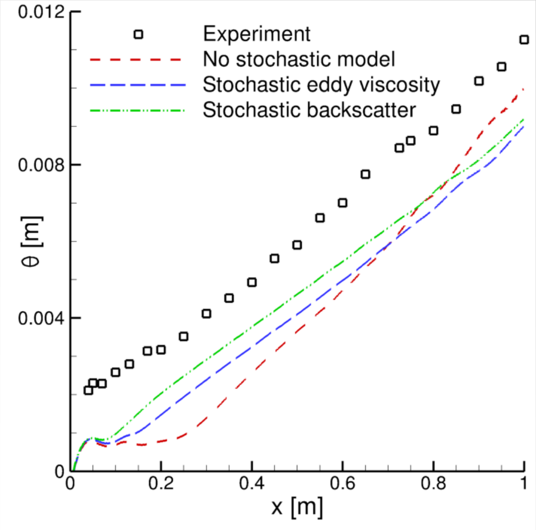
Spatial shear layer: Stochastic backscatter model



Baseline
stochastic eddy viscosity
(+ HPF)



Stochastic
backscatter model
(+ HPF)

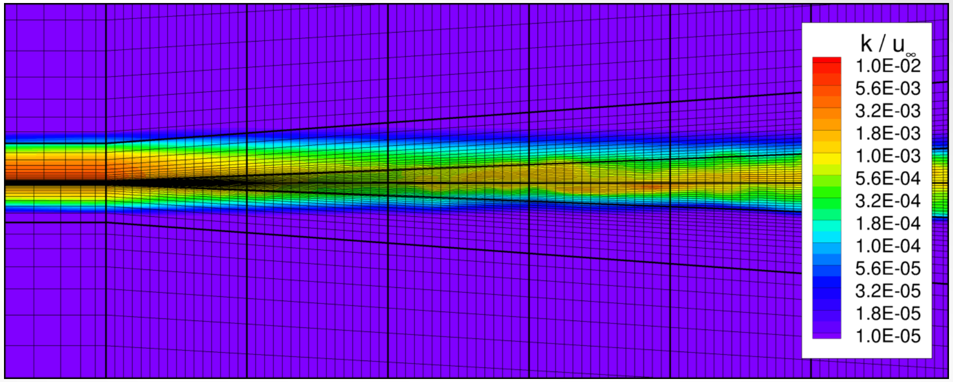


Momentum
thickness

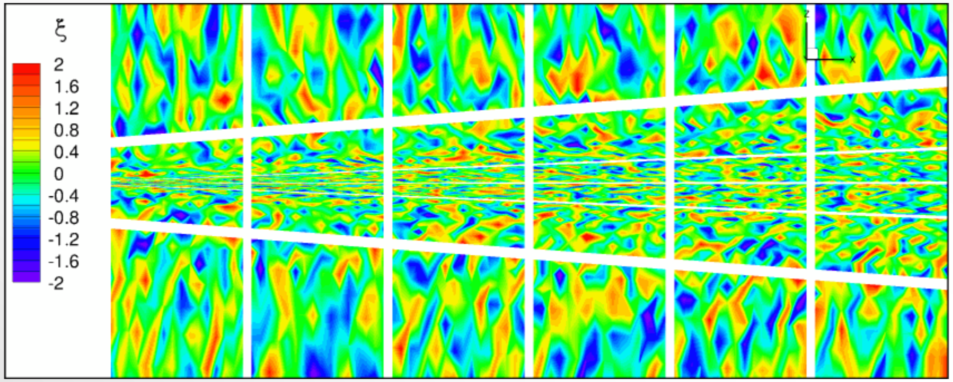
Triggering of instabilities

- Existing stochastic model
  - ξ drawn independently at every grid point and at every time step
  - less effective if high aspect ratios of grid cells ($\delta x \ll \Delta$) or if time step smaller than subgrid time scale ($\delta t \ll \Delta/\sqrt{k}$)

subgrid kinetic energy k



uncorrelated stochastic variable ξ



Triggering of instabilities

- Introduce spatial and temporal correlation
  - reasonable to assume stochastic variables to be correlated if $\delta x < \Delta$ or $\delta t < \Delta/\sqrt{k}$
  - spatial correlation: implicit filtering

$$\left(I - \beta_i \delta_i^2\right)\left(I - \beta_j \delta_j^2\right)\left(I - \beta_k \delta_k^2\right) \xi = \zeta \quad , \quad \beta_i = C_\Delta (\Delta/\delta_i x)^2$$

ζ = uncorrelated
ξ = spatially correlated

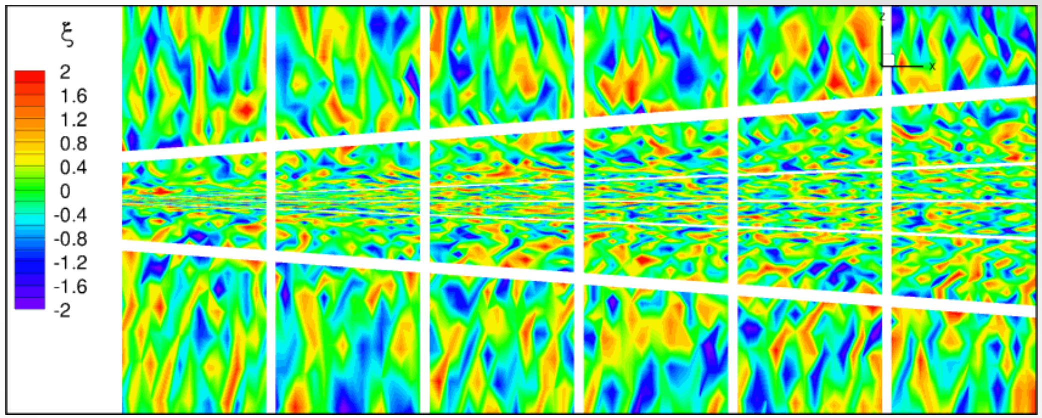
  - temporal correlation: stochastic differential equation (Schumann)

$$\rho\xi \mathrm{d}t + \left(\frac{\partial \rho\xi}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \xi)\right) \tau \, \mathrm{d}t = \sqrt{2\tau \, \mathrm{d}t} \, \rho\eta \quad , \quad \tau = C_\tau \Delta/\sqrt{k}$$

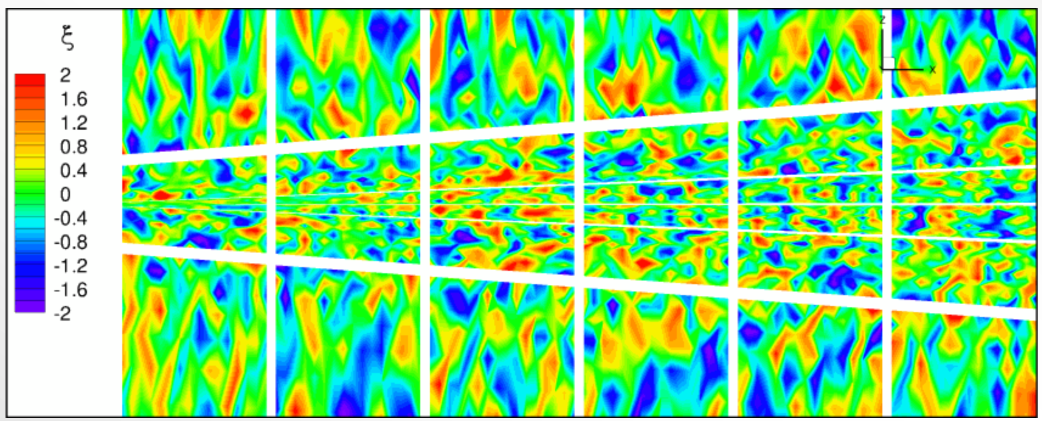
η = only spatially correlated
ξ = spatially and temporarlly correlated

Stochastic backscatter model with spatial correlation

uncorrelated stochastic variable ξ



spatially correlated stochastic variable ξ

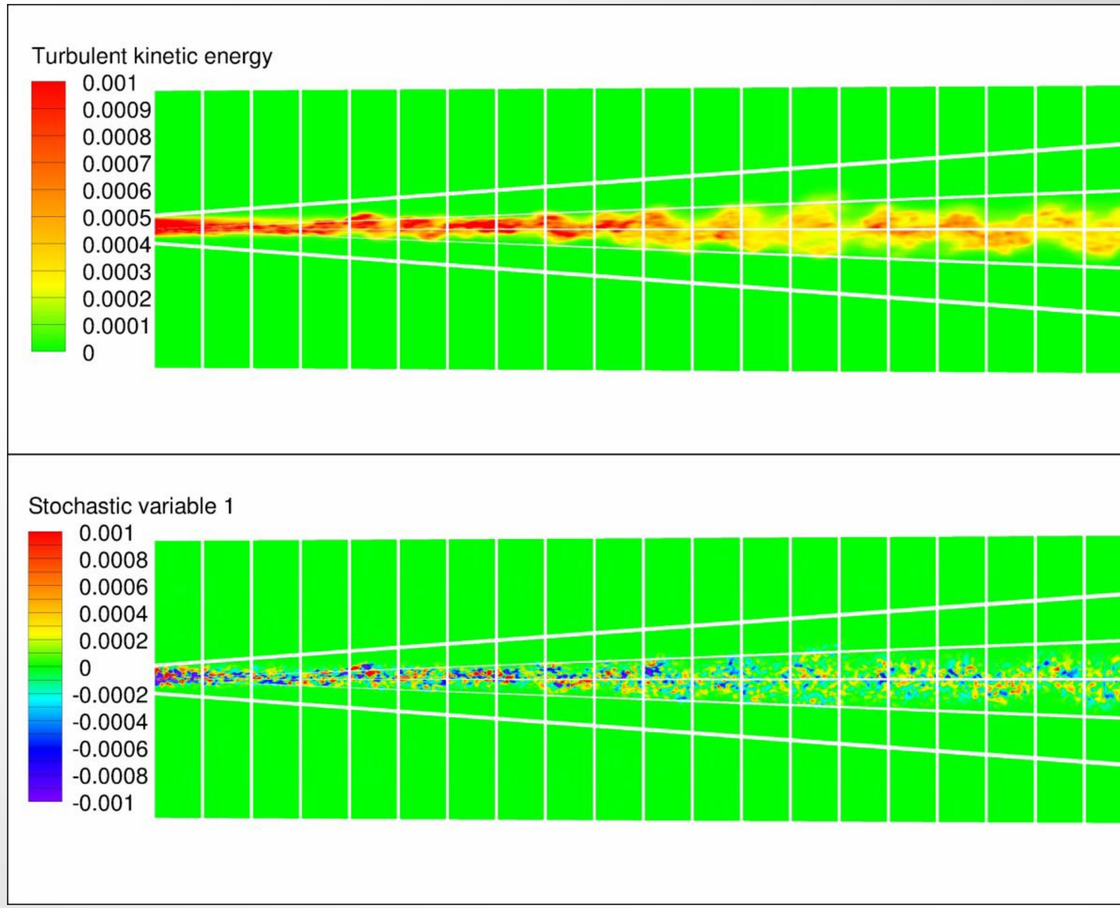


NLR

Stochastic backscatter model with spatial & temporal correlations

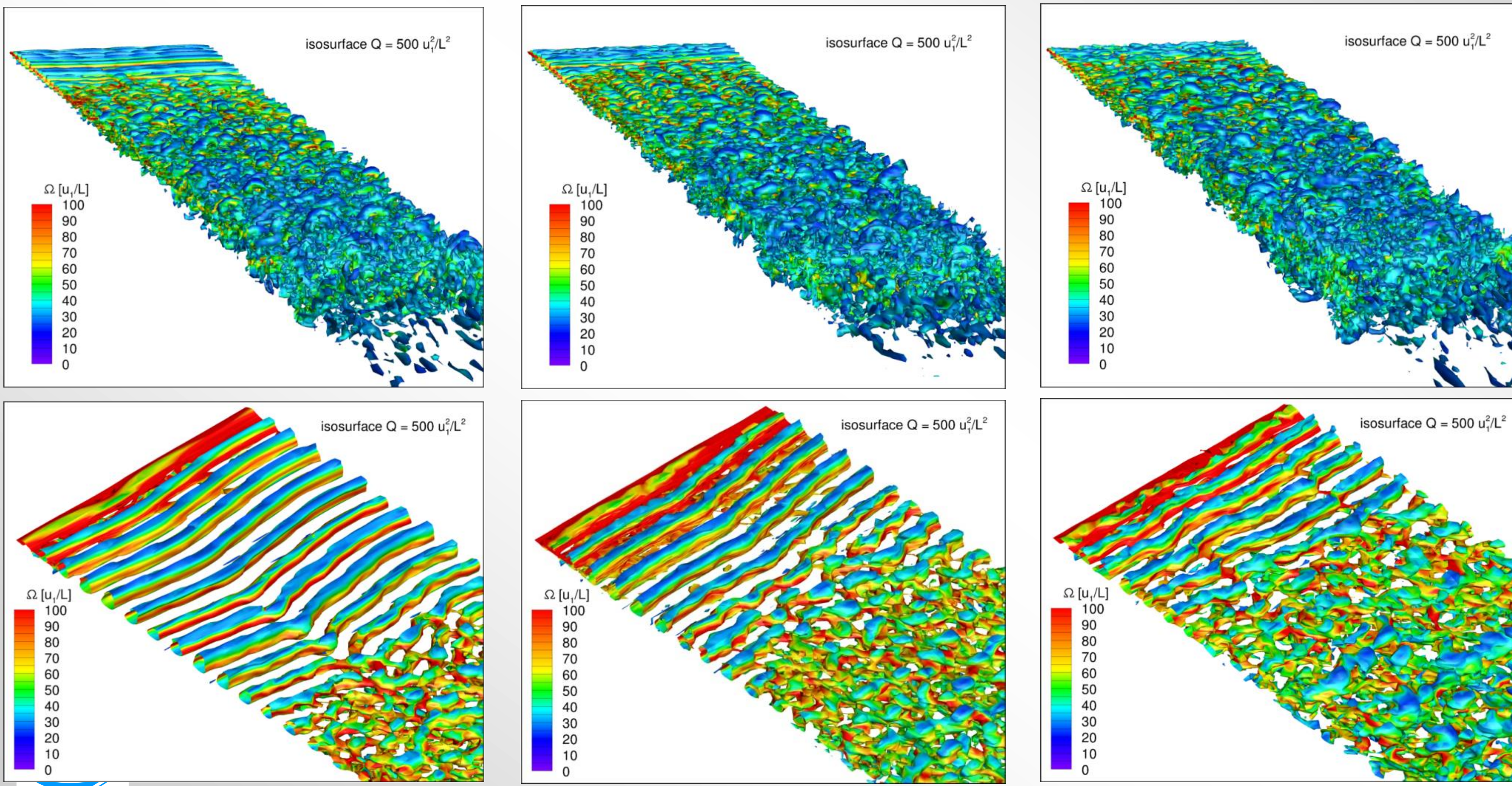
subgrid
kinetic energy
k

spatially and temporally
correlated
stochastic variable
$k\xi$

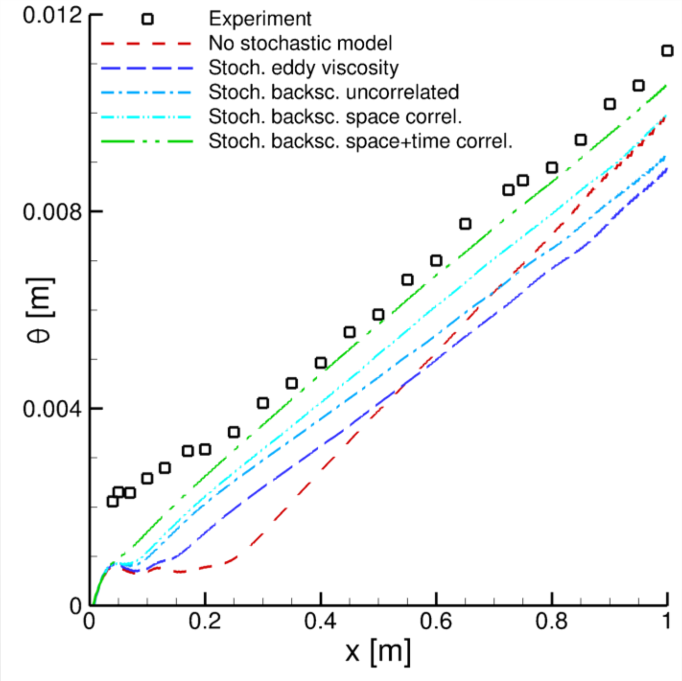


Spatial shear layer: Stochastic models

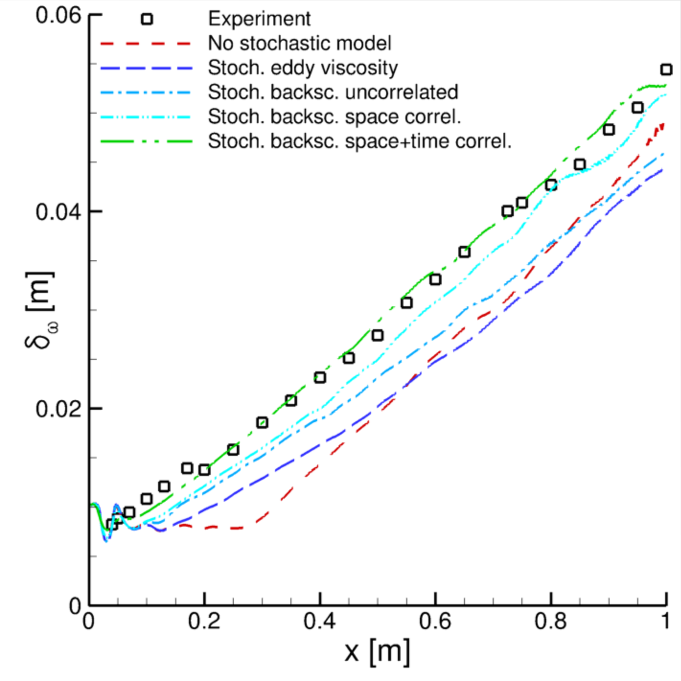
stochastic eddy viscosity | stochastic backscatter uncorrelated | stochastic backscatter temporal and spatial correlation



Spatial shear layer: Stochastic models

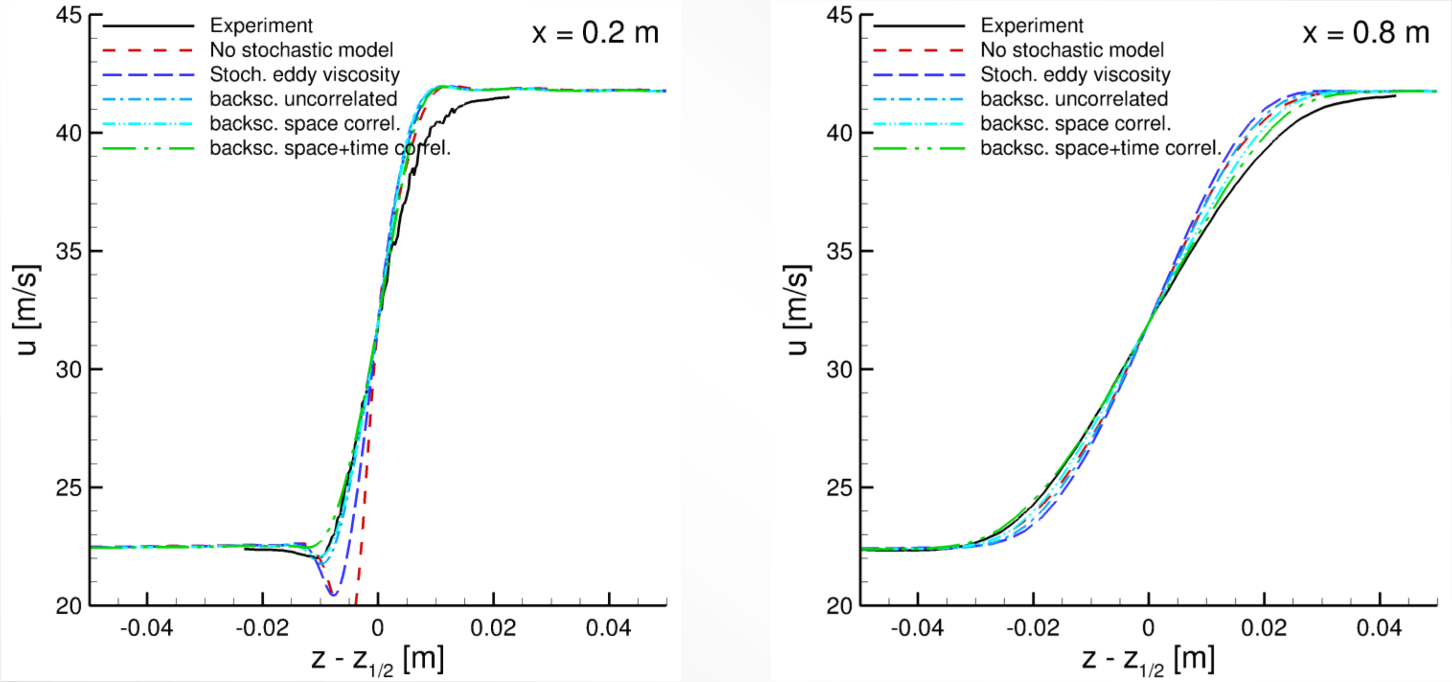


Momentum thickness



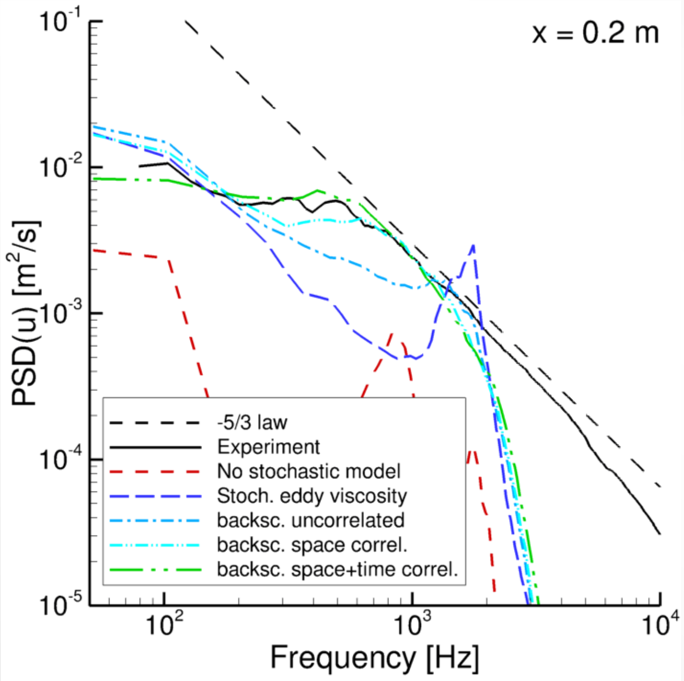
Vorticity thickness

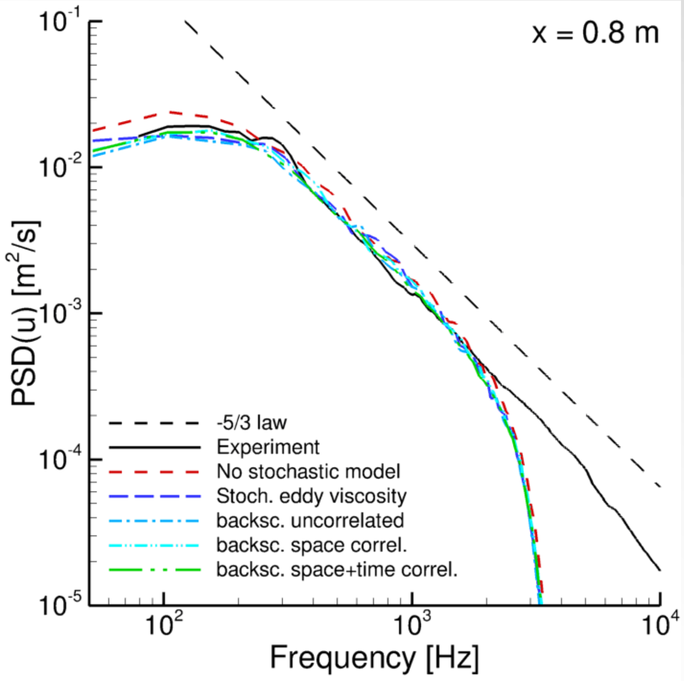
Spatial shear layer: Stochastic models



Velocity profiles

Spatial shear layer: Stochastic models

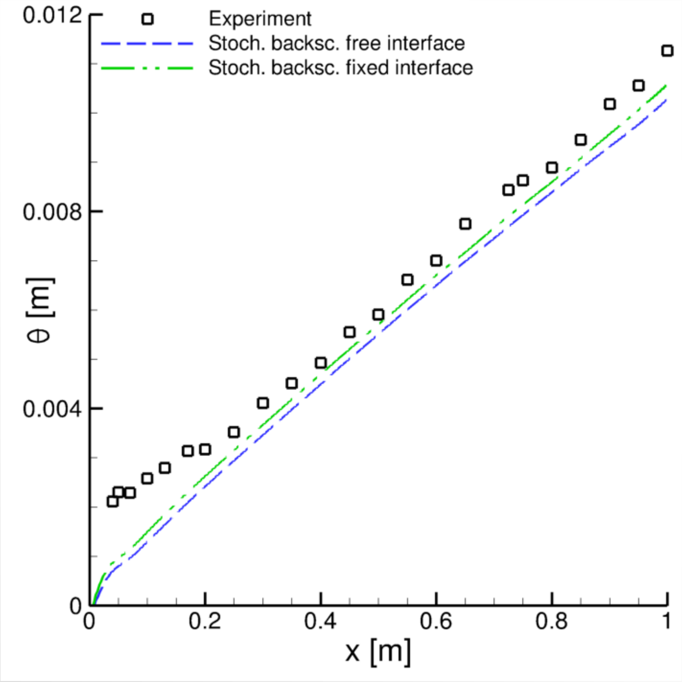




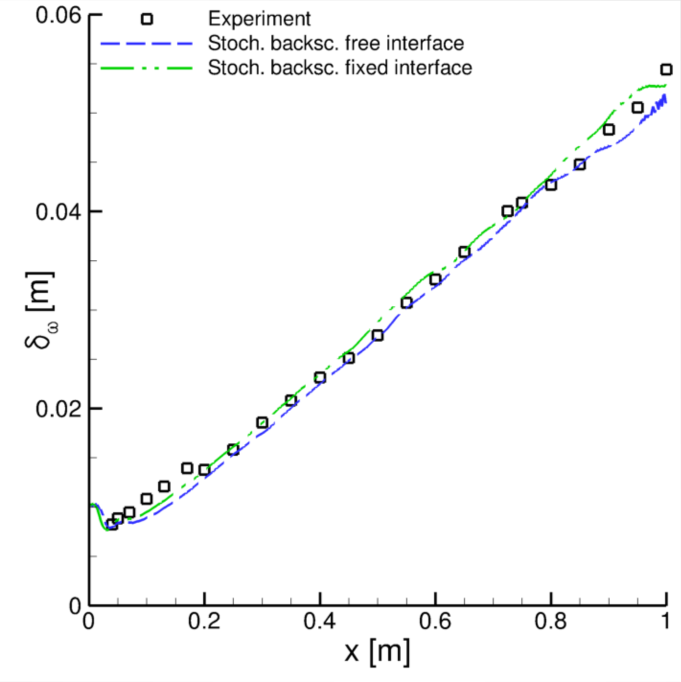
Energy spectra

Spatial shear layer: Stochastic models

- fixed vs. free RANS-LES interface (fully non-zonal)



Momentum thickness



Vorticity thickness

Conclusion

- Alternative methods to reduce subgrid stresses
  - Butterworth HPF equivalent to reference
  - Vreman model is ineffective
  - Nicoud σ model slighty better than reference
- Alternative stochastic model
  - stochastic backscatter model with temporal and spatial correlation gives strong improvement

Future work

- Testing other combinations of GAM approaches? (Task 2.1)
  - Nicoud σ model + stochastic backscatter model?
- Testing best approach for complex test cases (Task 2.2)
  - round jet
  - 3-element airfoil